Application of a Symbolic Probability Theory to Qualitative Reasoning under Uncertain Information

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Abstract

This paper deals with the Representation and the Symbolic Management of Uncertain information. After a presentation of three examples, we give the outlines of our theory, its axiomatics and its application rules, before ending with a discussion about the size of the the model and its influence on the results. The Symbolic Model presented here uses a M-valued Logic and a Symbolic Theory. The natural concepts of Uncertainty and Conditional Statement are represented with the aid of symbolic generalizations of the classical notions of Probability and Conditional Probability. For inferential processes, we use symbolic extensions of classical inferential deductive and probabilistic processes. The results given here show that such a Symbolic Theory constitutes a satisfying model of explicit management of Uncertainty of natural Language encoded in a qualitative way.

1. INTRODUCTION

We present here some examples of Qualititative Reasoning using a Symbolic Probability Theory. The approach developped here is Qualitative ; thus, no Numerical computing will be found. During the last ten years, a new concept, viewed as a Symbolic extension of the classical concept of Probability has been studied in order to represent symbolically the Uncertainty in the Bayesian Networks or the Uncertainty of statements of the natural Language. It has already lead to the elaboration of many Symbolic Theories (Cf. [Pear88(a), (b), [Alel88,90], [XBPo90], [HaFa92], [ADGP92], [DaGi92], [Darw92], [Pacd92(a), (b), 93], [Pool93], Pacd94(a), (b)]). The symbolic Theory used here has already been proposed by D. Pacholczyk (Cf. [Pacd93, Pacd94(a), (b), Pacd95]). Its semantic model is built upon a substrate of a M-valued Predicate Logic that has a better expressive power (Cf. [Pool91]) and allows a better account of the notion of Uncertainty. Moreover, it also allows the Management of Uncertainty and Vagueness, both expressed in a qualitative way. Its inferential processes may be of deductive type or of conditional type. This theory can also be used in the framework of a Symbolic Approach of Bayesian Networks (Cf. Pacd94(a), (b)]).

The adequation of this theory to the Representation and the Management of the Uncertainty of the statements of the natural Language expressed in a Qualitative way is also reviexed. Let's recall that the psychologists consider that the symbolic graduation can not be reasonnably apprehended by man above ten degrees (Cf. [LeNy87], [MaOv90], [RBGh90]). A first scale of degrees of Truth enables to express the graduation of the Vagueness. A second scale of Uncertainty degrees, distinct from the first, is used to express the graduation of the Uncertainty. Thus, stating that "it is rather-probable that Smith be a *very* rich man" is equivalent to say that "Smith is a *very* rich man" *satisfies* the predicate Cert to the degree rather. The underlying many-valued Logic satisfies a set of semantic axioms translating the minimal rules that govern the concept of Certainty. We then obtain a semantic model of the Uncertainty Management having both the representation of its graduation and the basic rules governing its management. All the definitions and all the results presented here will be translated with the aid of a certainty function of statements, called Cert. Thus, the statement: "it is rather-probable that Smith be a *very* man", whose formal translation is $\mathcal{A}\models_{\mathsf{rather}}$ $\mathtt{Prob}$(Smith is a *very* rich man) will be expressed under the following form : $\mathtt{Cert}$(Smith is a *very* rich man) = rather($\mathtt{Probable}$).

Section 2 will provide three examples written in Natural Language and their treatment using the formulas and rules from this Symbolic Theory. Section 3 will deal with the theoretical framework, that is, an enriched M-valued Logics. It is important to note that no example can be numerically approached because of the uncertainty of the knowledge. Therefore, our aim in Section 4 will be to discuss about the size of the model (5,7,9...) and the different possible values in a chosen model, showing that an overlapping ser ies of results can be found for any M.

2. THE EXAMPLES

2.1 CITATIONS IN TEXT

Citations within the text should include reference number between brackets, e.g [1].

2.2 FOOTNOTES

Indicate footnotes with a number[1] in the text. Place the footnotes at the bottom of the page on which they appear.

2.3 FIGURES

All artwork must be centered, neat, clean, and legible. Figure number and caption always appear below the figure. Each figure should be numbered consecutively.

Figure 1: Sample Figure Caption

2.4 TABLES

All tables must be centered, neat, clean, and legible. Do not use hand-drawn tables. Table number and title always appear above the table. See Table 1.

Table 1: Sample Table Title

PART	**DESCRIPTION**
Dendrite	Input terminal
Axon	Output terminal
Soma	Cell body (contains cell nucleus)

Acknowledgements

Use unnumbered headings for the acknowledgements. All acknowledgements go at the end of the paper.

[1] Sample of the first footnote

References

[1] G. Tesauro (1989). Neurogammon wins computer Olympiad. *Neural Computation* **1**(3):321-323.